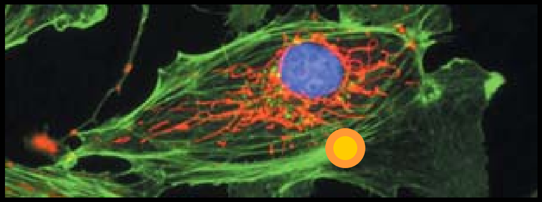
Microrheology of Complex Fluid

☐ Rheology: Science of the deformation & flow of matter

☐ Microrheology

- Microscopic scale samples
- Micrometer lengths

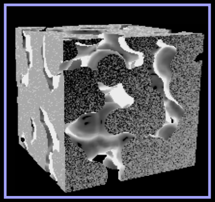


probes.com

Complex shear modulus G*(ω)

$$\boxed{\sigma = G^* \varepsilon}$$

- $G^*(\omega) = G'(\omega) + j\,G''(\omega)$
- Solid *vs.* fluid
- Resistance to deformation

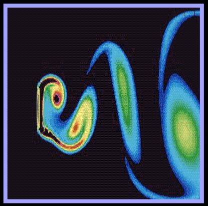


ciks.cbt.nist.gov

Storage modulus G'

Energy storage

Elasticity ~ Solid



ma.man.ac.uk

Loss modulus G''

Energy dissipation

Viscosity ~ Fluid

High Frequency Microrheology Measurement

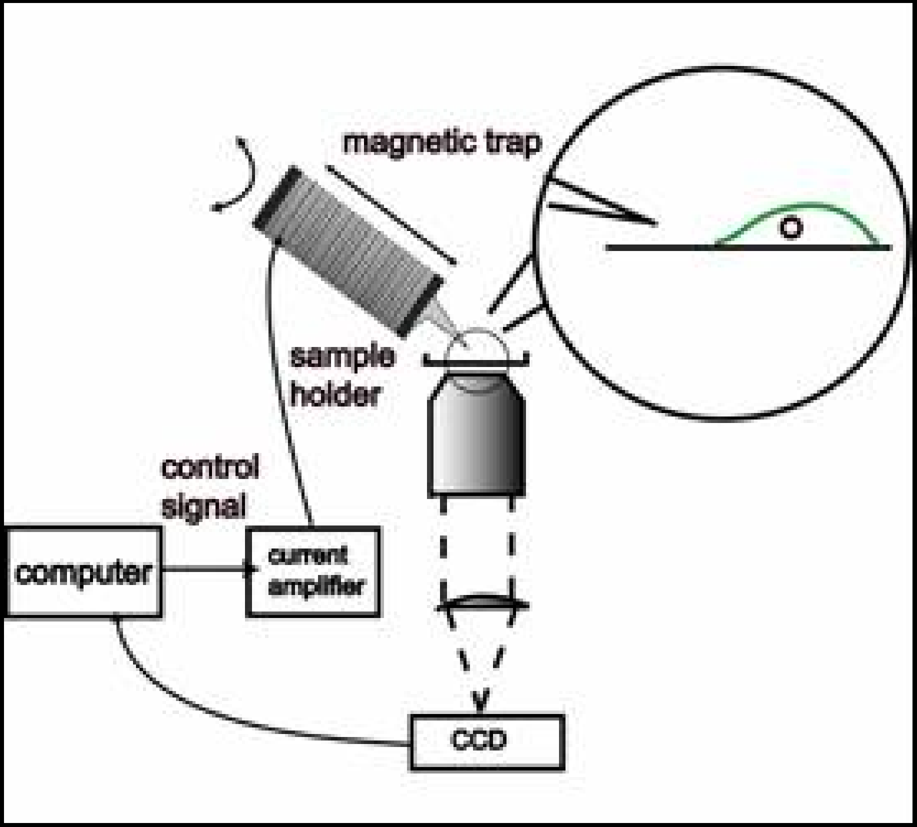
Active Method:

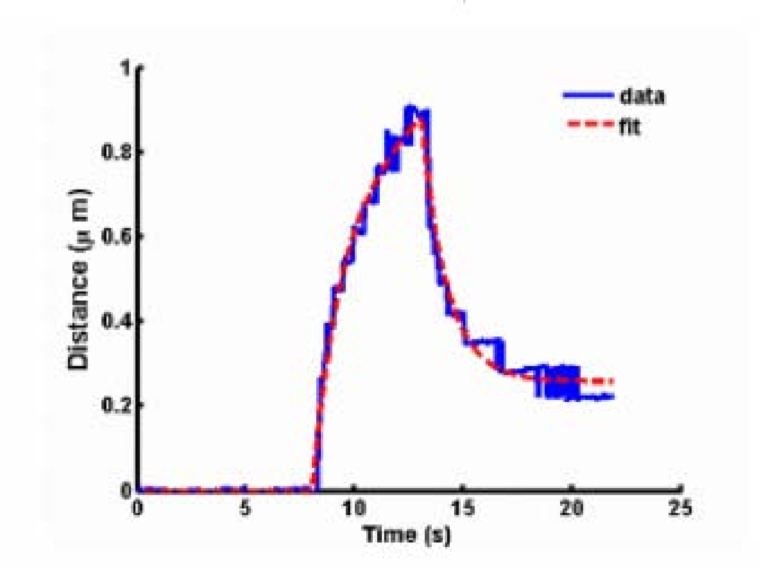
- Magnetic microrheometer – Baush, BJ 1998
  Huang, BJ 2002

Passive Method:

- Single particle tracking – Mason, PRL 1995
  Yamada, BJ 2000
- Multiple particle tracking – Crocker, PRL 2000

Magnetic Microrheology





Magnetic Microrheology

5 sec Step Response

Basic Physics of Magnetic Microrheometer

Ferromagnetic particle

$$\mathbf{F} = \frac{1}{2}\mu_0 \nabla(\mathbf{m}\cdot\mathbf{H})$$

Particles cluster together!
Doesn't work!

Paramagnetic particle – no permanent magnetic moment

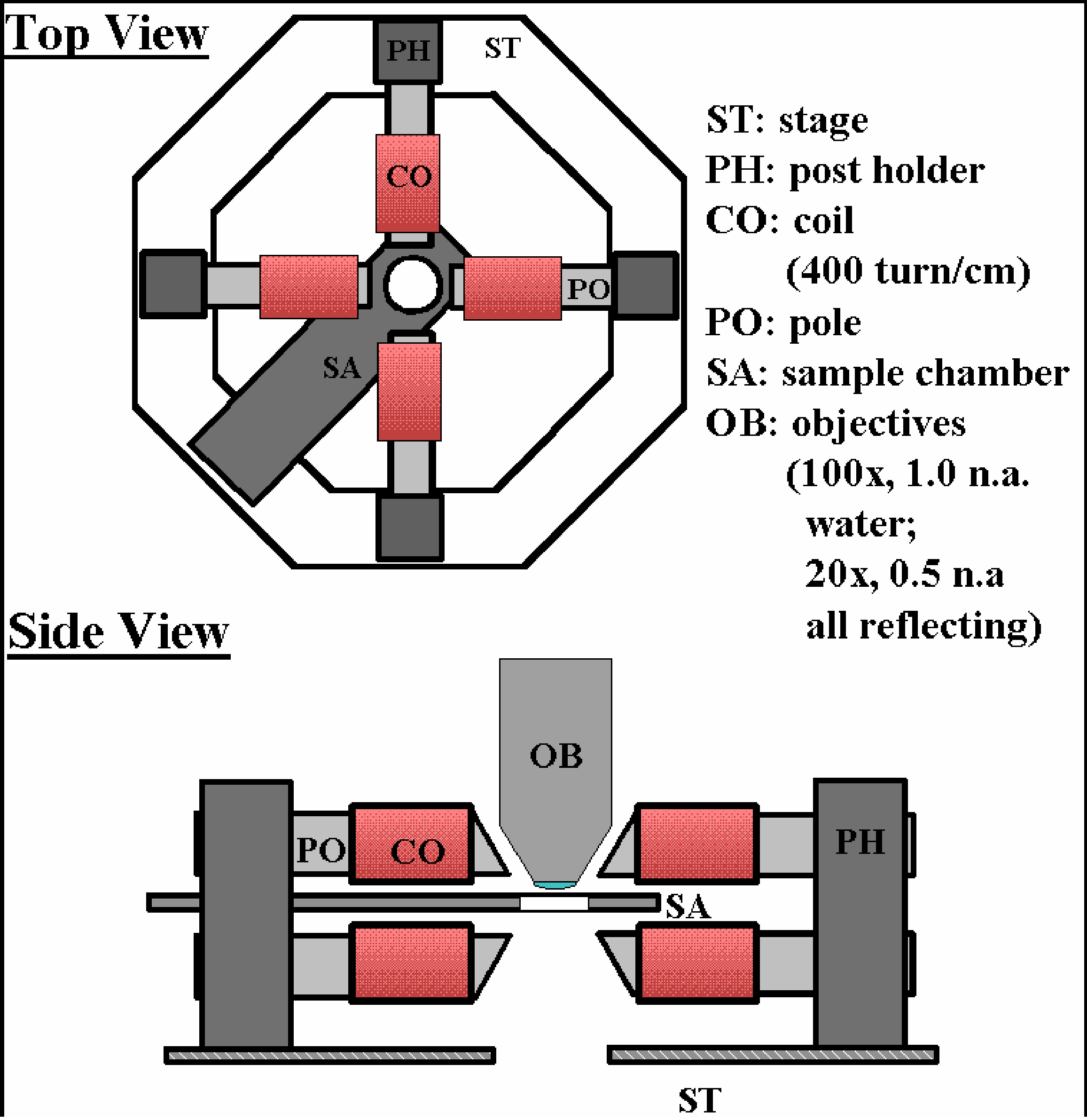
$$\mathbf{F} = \mu_0 \chi V \nabla(\mathbf{H}\cdot\mathbf{H})$$

χ is suceptibility

V is volume

Note: (1) force depends on volume of particle
(5 micron bead provide 125x more force)
(2) force depends on magnetic field GRADIENT

Magnetic manipulation in 3D



ST: stage
PH: post holder
CO: coil
(400 turn/cm)
PO: pole
SA: sample chamber
OB: objectives
(100x, 1.0 n.a.
water;
20x, 0.5 n.a
all reflecting)

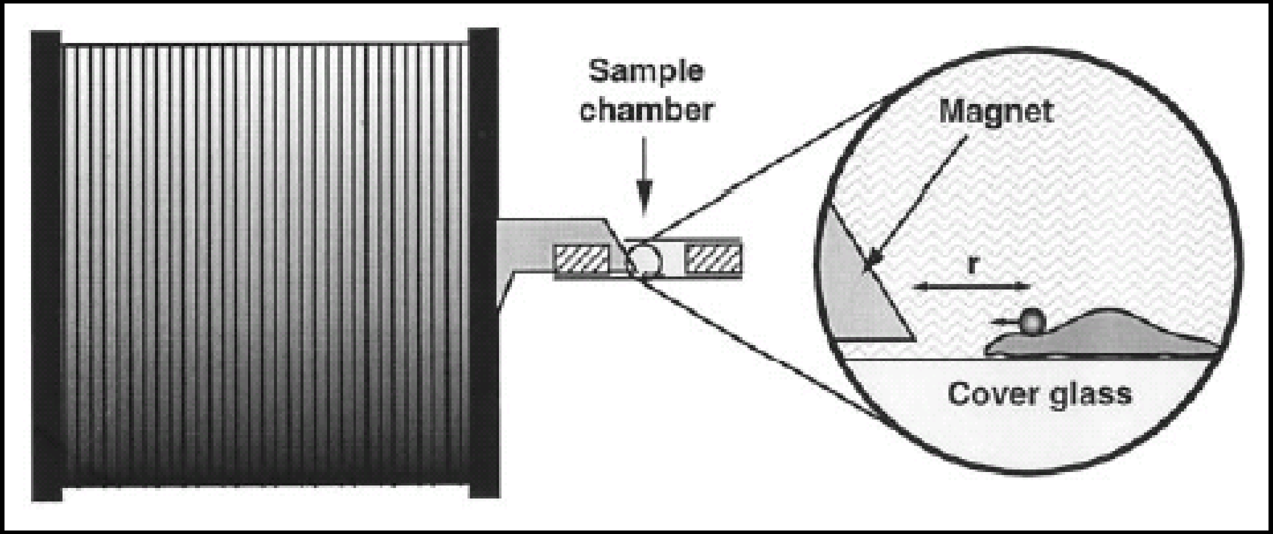
*Lower Force
nN level

*3D

*Uniform gradien

Amblad, RSI 1996
Huang, BJ 2002

Magnetic manipulation in 1D



*High force
 >10 nN

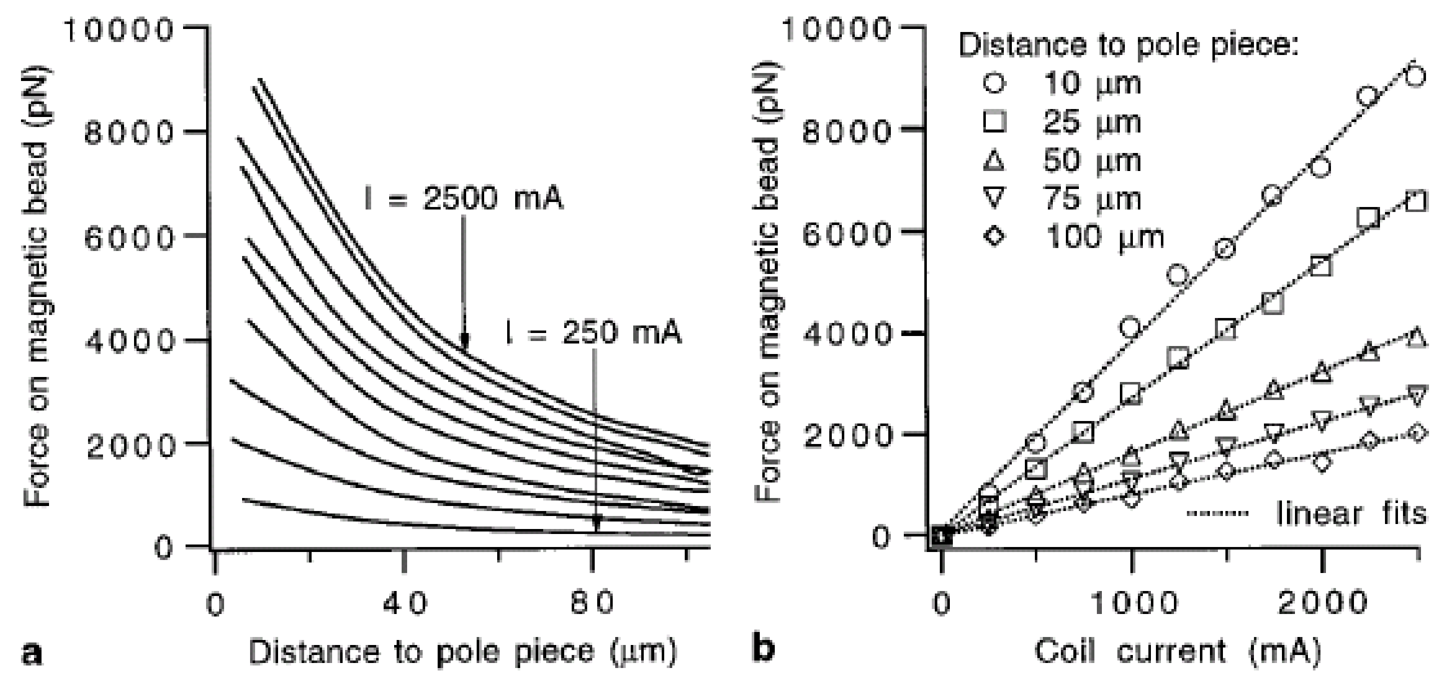
*Field non-uniform
 Needs careful
 alignment of tip
 to within microns

*1D

Baush, BJ 1998

The bandwidth of ALL magnetic microrheometer is limited by the inductance of the eletromagnet to about kiloHertz

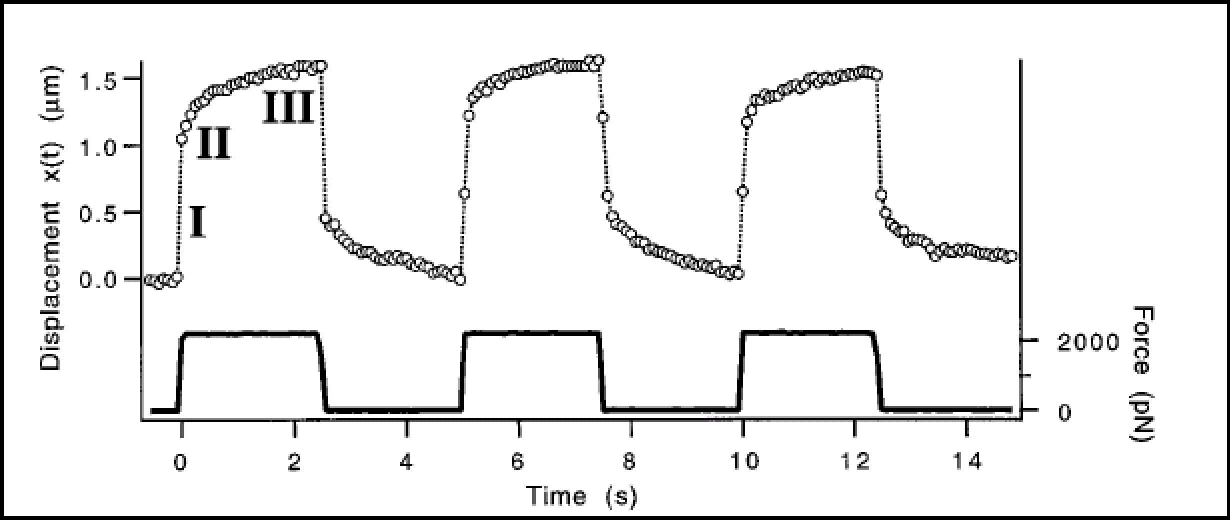
Magnetic Rheometer Requires Calibration

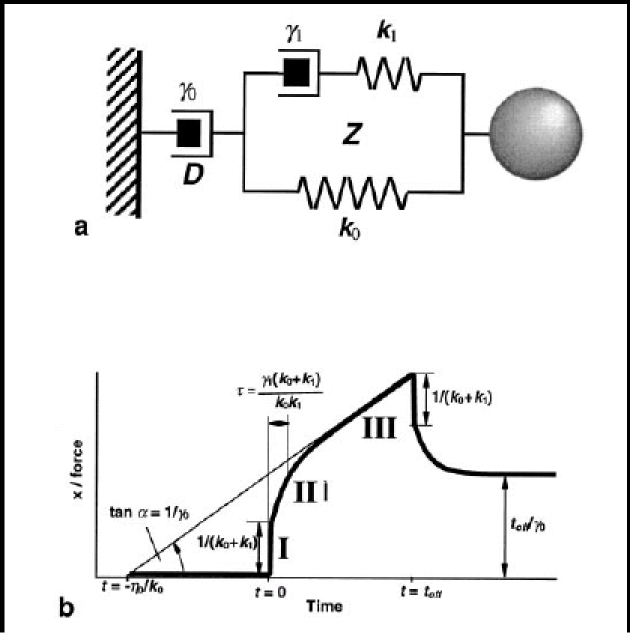


Baush, BJ 1998

Mag Rheometer Experimental Results

Baush, BJ
1998



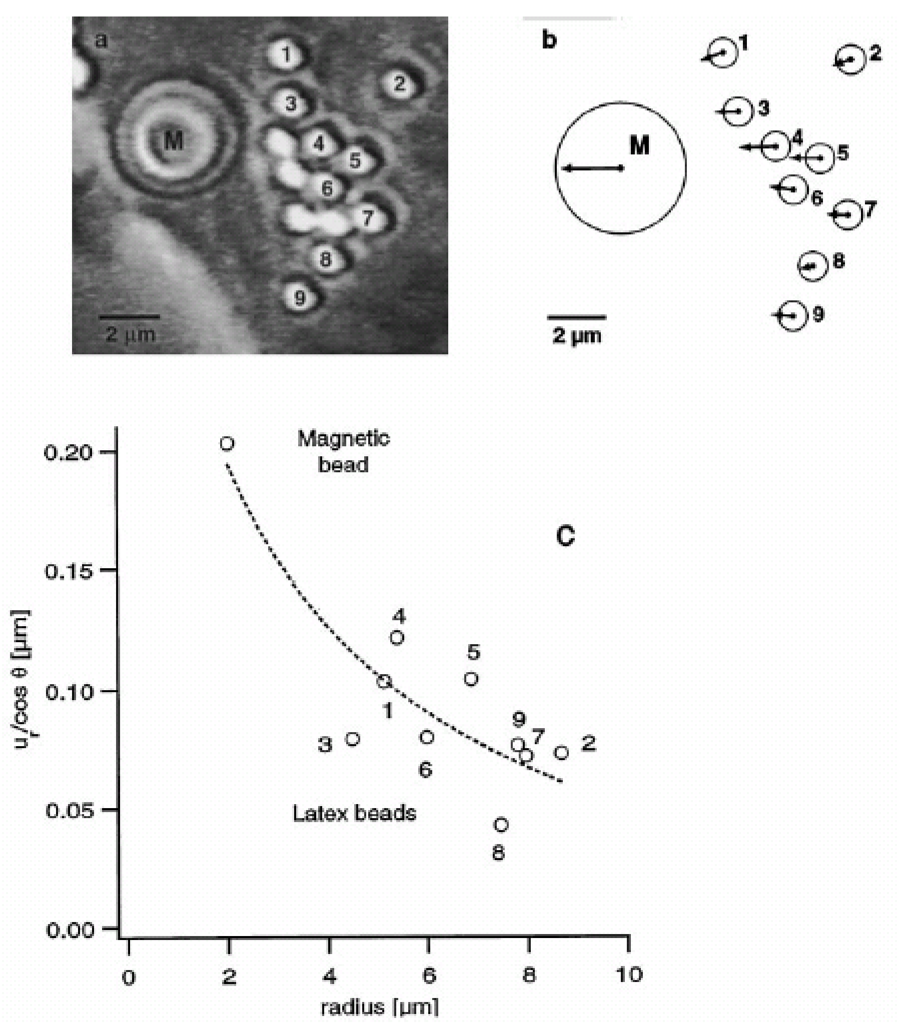


Transient responses allow fitting to micro-mechanical model

Problem – Magnetic bead rolling

Solution – Injection, Endocytosis
Modeling (Karcher BJ 2003)

Model Strain Field Distribution



Baush, BJ 1998

Single Particle Tracking

Consider the thermal driven motion of a sphere in a complex fluid

Langevin Equation

$$m\dot{v}(t) = f(t) + \int_0^t \xi(t-t')v(t')dt'$$

Inertial
force

Random
thermal
force

Memory function—
Material viscosity
Particle shape

Langevin Equation in Frequency Domain

Laplace transform of Langevin Equation

$$\tilde{v}(s) = \frac{\tilde{f}(s) + mv(0)}{\tilde{\xi}(s) + ms}$$

Multiple by v(0),
taking a time average,
Ignoring inertial term

$$\tilde{G}(s) = \frac{kT}{\pi a s < \Delta\tilde{r}^2(s) >}$$

Random force

$$< \tilde{f}(s)v(0) >= 0$$

Equipartition of energy

$$m < v(0)v(0) >= kT$$

Generalized Stokes Einstein

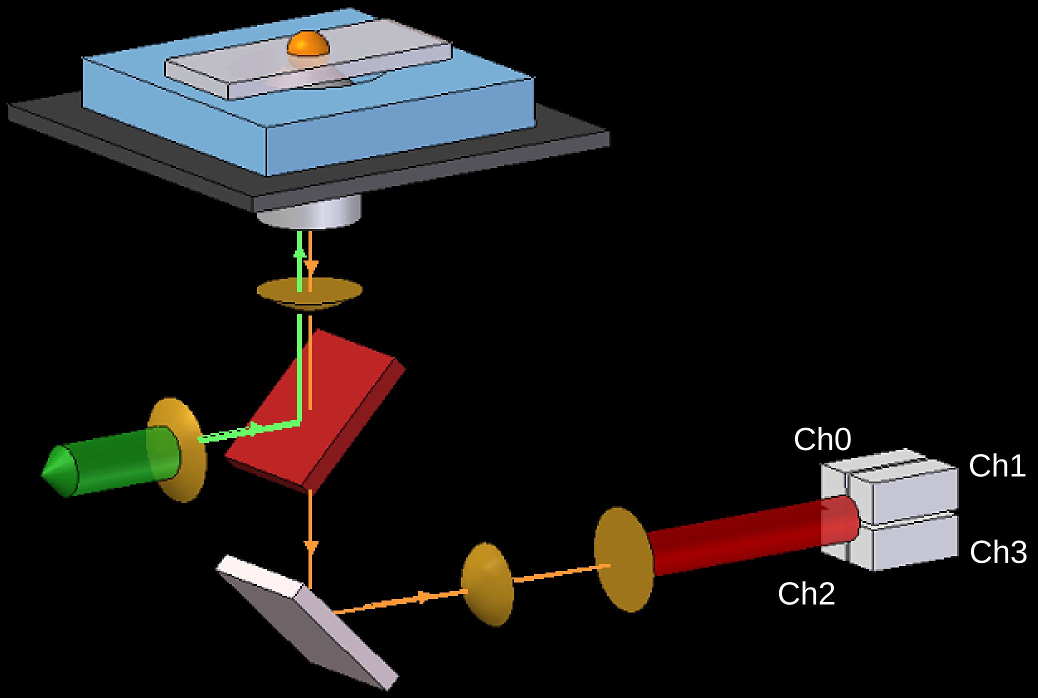
$$\xi(s) = 6\pi a\tilde{\eta}(s) \qquad \tilde{G}(s) = s\tilde{\eta}(s)$$

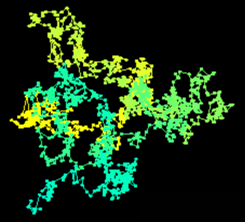
Definition and Laplace transform
of mean square displacement

$$< v(0)\tilde{v}(s) >= s^2 < \Delta\tilde{r}^2(s) > / 6$$

(2) Fluorescence Laser Tracking Microrheometer

- Approach: Monitoring the Brownian dynamics of particles embedded in a viscoelastic material to probe its frequency-dependent rheology





trajectory

mean squared displacement

$$\left\langle \Delta R^2(\tau)\right\rangle = \left\langle \left(\vec{r}(t+\tau)-\vec{r}(t)\right)^2\right\rangle$$

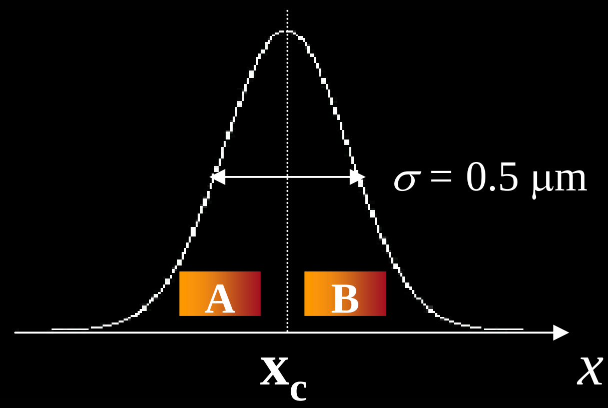
shear modulus

$$G^*(i\omega)=\frac{2k_BT}{3\pi\cdot a\cdot i\omega\cdot\left\langle \Delta\tilde{R}^2(i\omega)\right\rangle}$$

Yamada, Wirtz, Kuo, *Biophys. J.* 2000

(2) Nanometer Resolution for the Bead’s Trajectory

- Collecting enough light from a fluorescent bead is critical



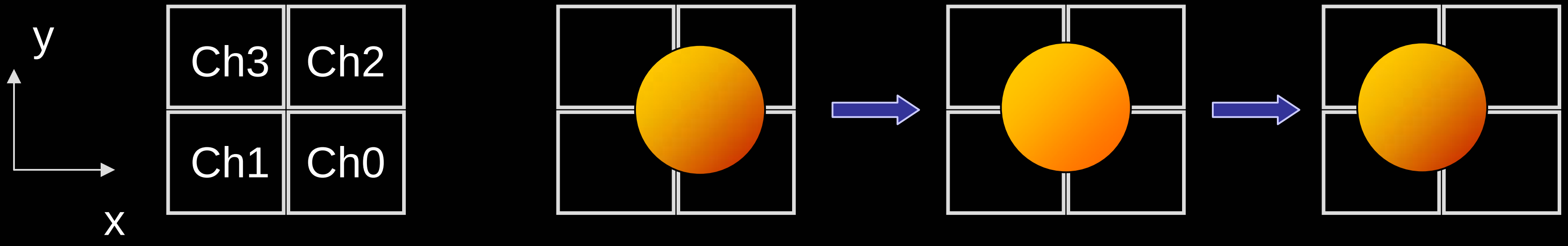
- $$p(x)=\frac{1}{\sigma\sqrt{2\pi}}\exp\left[-\frac{(x-x_c)^2}{2\sigma^2}\right]$$

- $$\frac{N_A}{N_B}=\frac{p(A)}{p(B)}=\frac{\int_{-\infty}^{x_c}\exp\left(\frac{-x^2}{2\sigma^2}\right)dx}{\int_{x_c}^{+\infty}\exp\left(\frac{-x^2}{2\sigma^2}\right)dx}$$

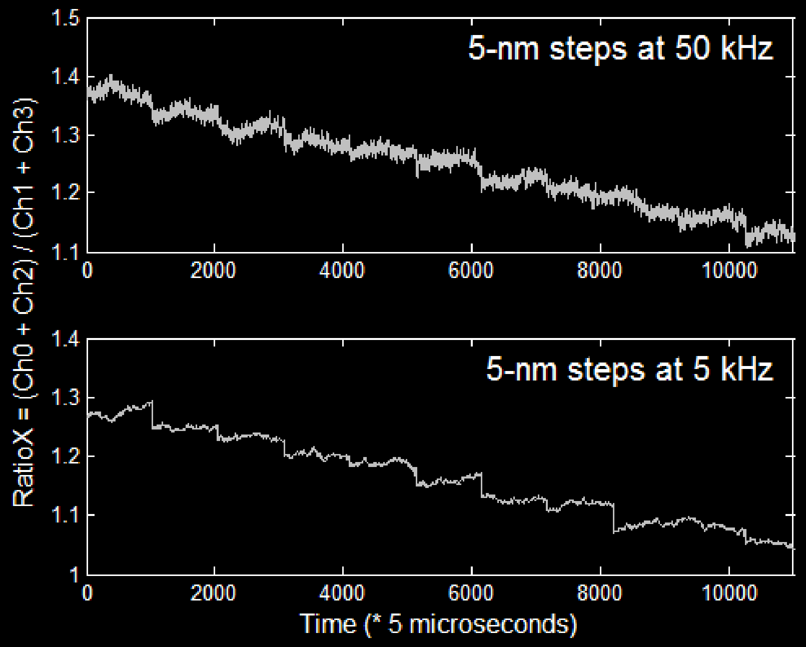
Photons detected per measurement	10^3	10^4	10^5	10^6
Uncertainty on $\frac{N_A}{N_B}$	0.033	0.010	0.003	0.001
Uncertainty on x_c (nm)	12	4	1.2	0.4

Nanometer resolution ↔ 10^4 photons per measurement

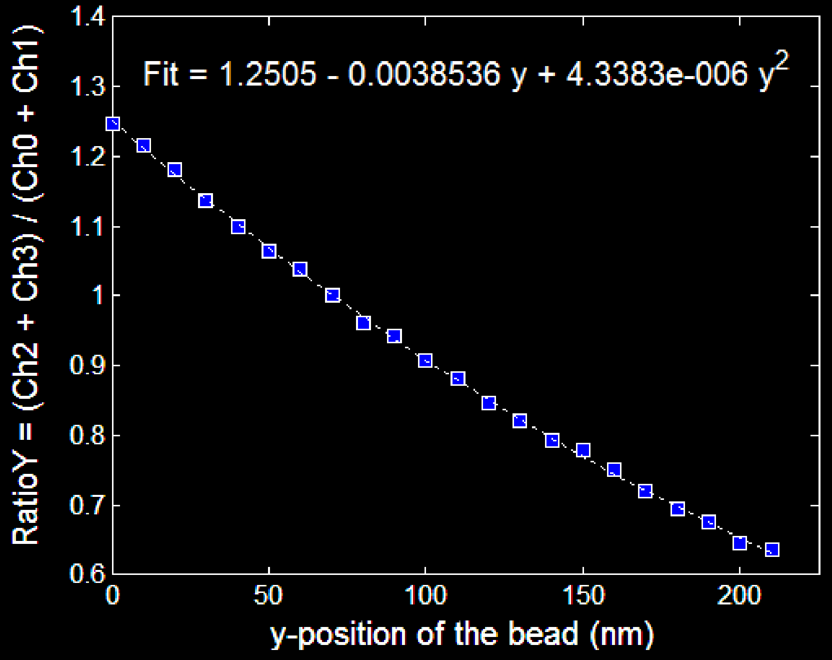
(2) Calibrating the FLTM



- 5-nm stepping at 5 or 50 kHz



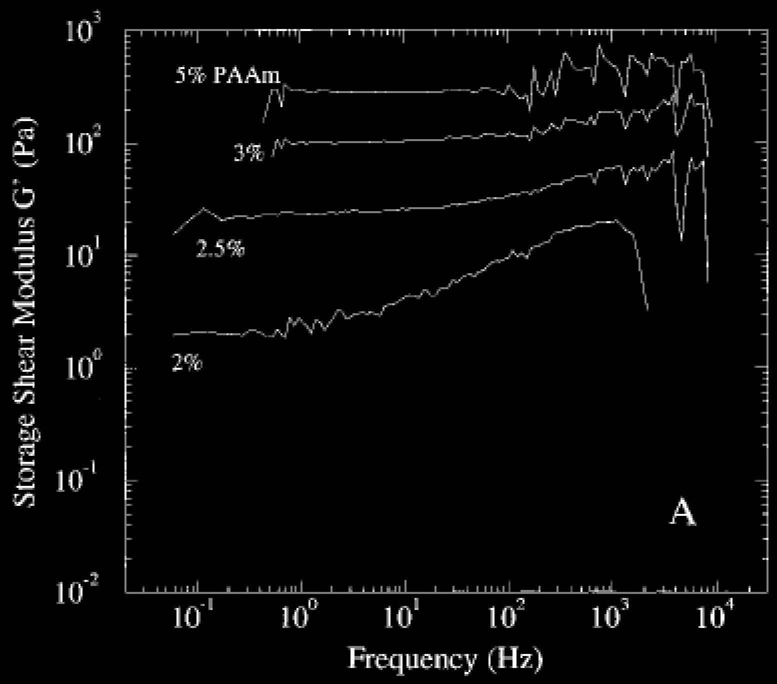
- Curve fitting matches theory

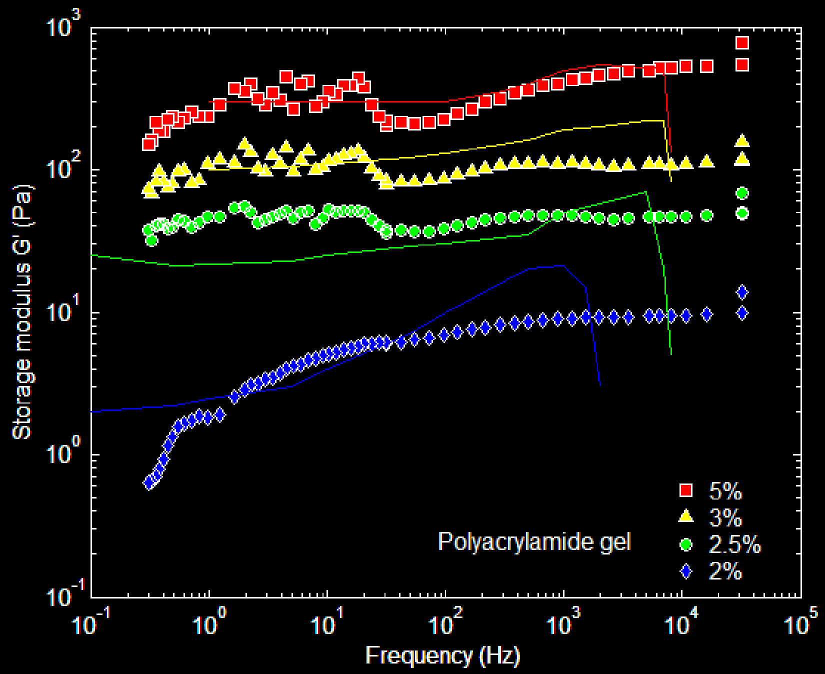


Characterizing the FLTM

- Using polyacrylamide gels (w/v 2% to 5%) of known properties

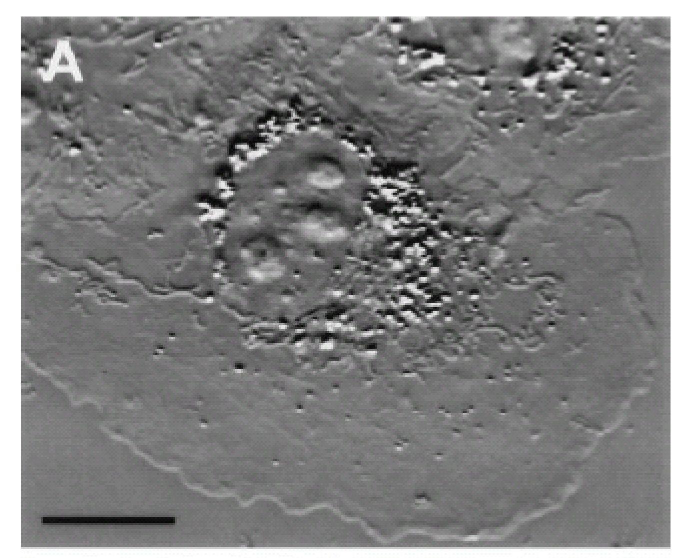
✓ Good agreement with previously published data

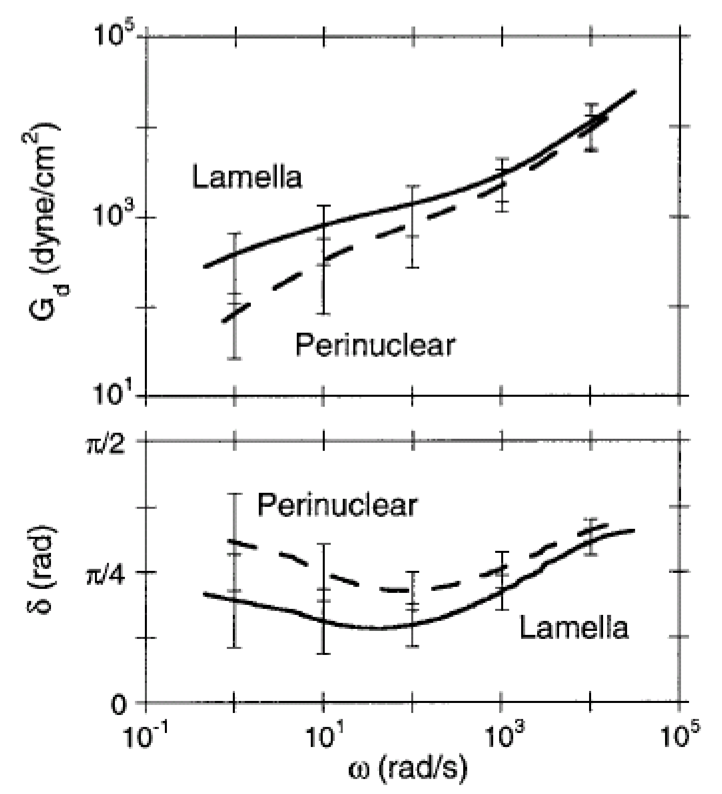




Schnurr B., Gittes F., MacKintosh F.C. & Schmidt C.F
Macromolecules (1997), 30, p.7781-7792

Single Particle Tracking Data





Yamada BJ 2000

Two- and Multiple Particle Tracking

SPT responses can be influence by local processes (adhesion, active, etc) and not represents global cytoskeleton behavior

Solution: Look at the correlated motion of two particles under thermal force

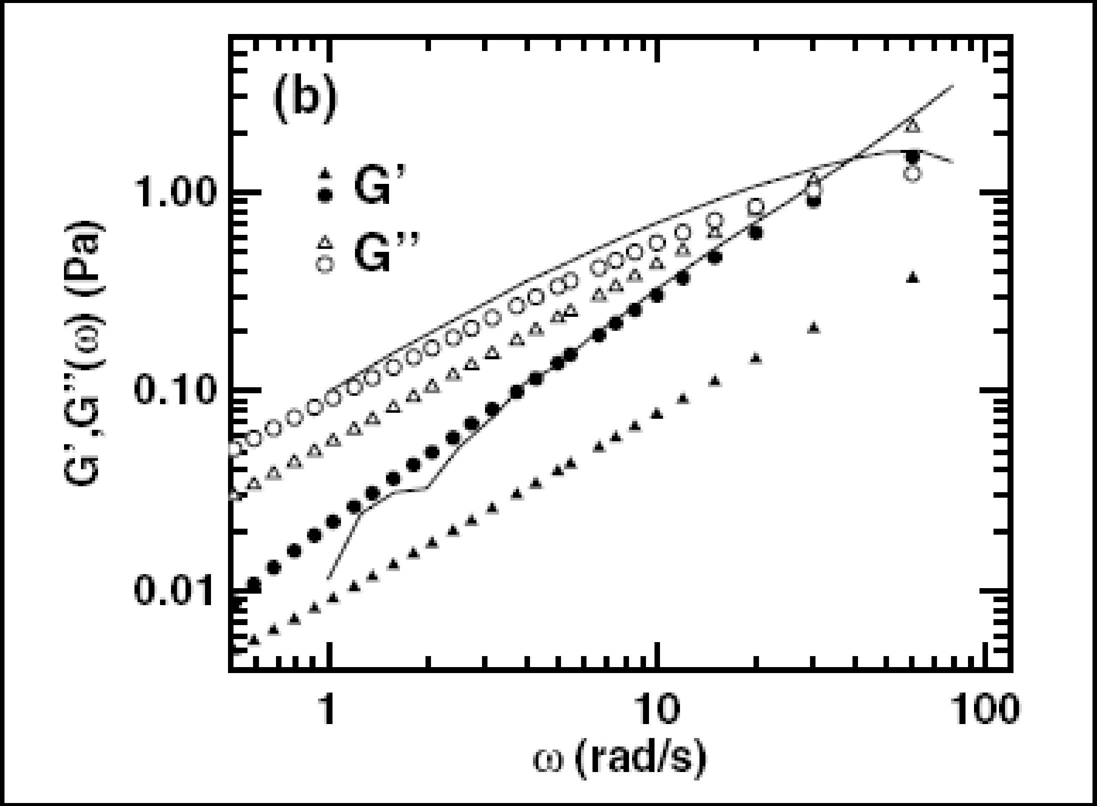
$$D_{rr}(r,\tau) = < \Delta r_r^i(t,\tau)\Delta r_r^j(t,\tau)\delta(r - R^{ij}(t)) >_{i \neq j,t}$$

$$D_{rr}(r,s) = \frac{kT}{2\pi r s \tilde{G}(s)}$$

The major difference is that the correlation signal is a function of “r” the separation of the particles but not their size

Instead of using fast quadrant detectors, multiple particle tracking uses a wide field camera which is slower

SPT vs MPT



Triangle: SPT

Circle: MPT

SPT and MPT results can be quite different specially in cells

Crocker, PRL 2000

A Comparison of Microrheometry Methods

	Magnetic	SPT	MPT
Bandwidth	kHz	MHz	kHz
Signal Amplitude	μm	nm	nm
Local Effects	Yes	Yes	No
Nonlinear regime	Yes	No	No
Instrument	Intermediate	Intermediate	Simple