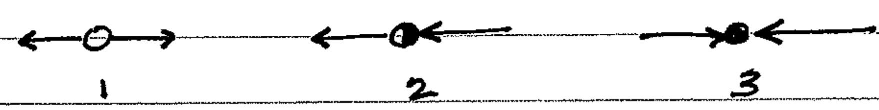
2·13·04

Defining Stability

so far stability has been defined by small pertebations.
do you travel away ~~fast~~ from ~~or~~ the fixed point w/ a small pertebation.
OR do you return to the same fixed point.

this was for 1D systems (i.e. flow on the line).

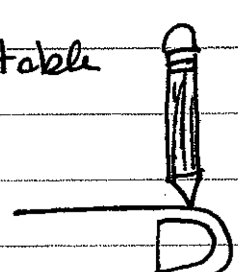


three possibilities : 3. stable fixed point
1. unstable fixed point
2. mixed stability fixed point.
4. or blow up

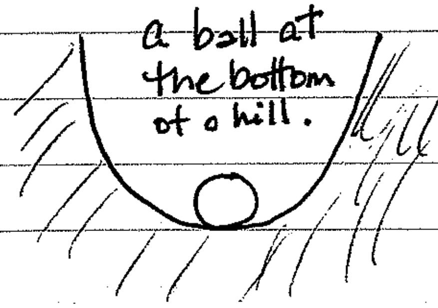
But for 2D systems, we can flow/maneuver through a plane, so we have a wider range of possible behaviors.

conceptual analogy of unstable + stable fixed point

unstable



pencil
perfectly balanced
on your finger



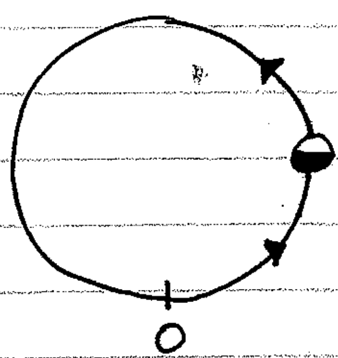
a ball at the bottom of a hill.

<u>Attracting</u> : a fixed point is attracting if all trajectories starting near the point approach the point as time $\to \infty$.

i.e. given enough time you will eventually end up at the fixed point if you start anywhere <u>near</u> it.

- says nothing about the path you take to get there.
- ~~where you end up~~
- so you could travel away from the point before you approach it again.

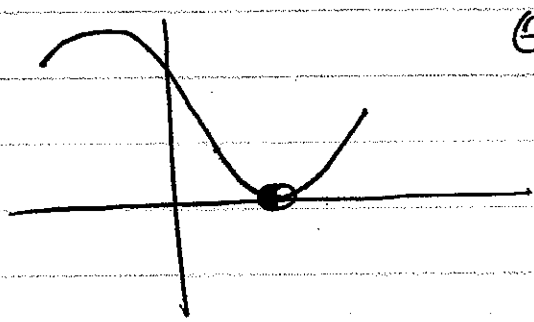
ex.



our fixed point is attracting even though you may travel away from your point

remember our flow on the circle - nonuniform oscillator

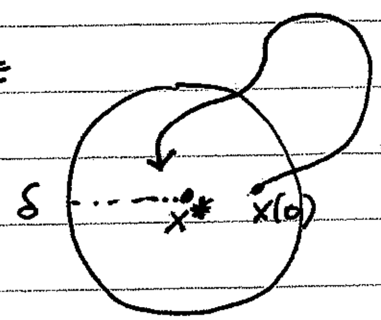
$$\dot{\theta} = 1 - \sin\theta$$

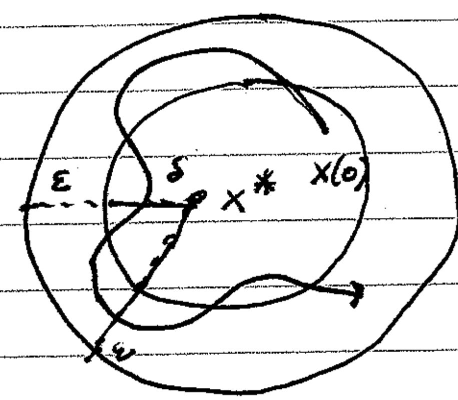


<u>Liapunov Stable:</u> a fixed point is Liapunov
stable if all trajectories that
start close to (x^*) the fixed point
~~tt~~ you will stay close to x^* for <u>all time</u>

- for all time : it does ~~e~~ say something
about the path. it must stay near ;
it cannot stray too far away.

- also again near close must be defined.





attracting & Liapunov stable
one can exist w/o the other.

our oscillator example is
attracting but not Liapunov
stable.

the term stable implies both
attracting & Liapunov stable.

we didn't have to differentiate for flows on
a line

unstable means : neither attracting
nor Liapunov stable.